Time-headways for interacting particle systems in stationary state

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ÚTIA AS
8th September 2014

Introduction

Scope of the talk

- interacting particle systems as traffic flow models
- headway distributions in IPS

Questions to be answered

- distance-headway distribution:
- time-headway distribution

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> "What is the probability that there is a gap of the length n between two consecutive vehicles/particles?"

- time-headway distribution

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Questions to be answered

- distance-headway distribution:

  "What is the probability that there is a gap of the length n between two consecutive vehicles/particles?"

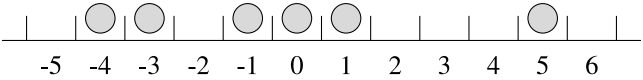
- time-headway distribution

  "What is the probability that there is a time interval of the length t between the passes of two consecutive vehicles/particles through a reference point?"

IPS in Exclusion Process representation

- at most one particle in site x
- state of x is τ_x

$$\tau_x = \begin{cases} 1 & x \text{ occupied,} \\ 0 & x \text{ empty,} \end{cases}$$



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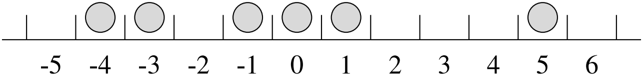

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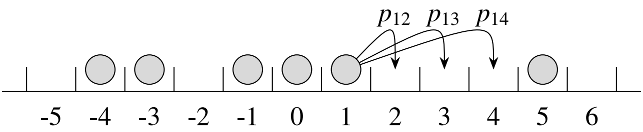
- $p(x,y)$ – underlying random walk



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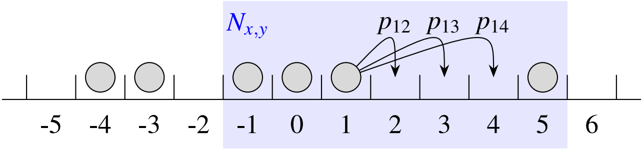
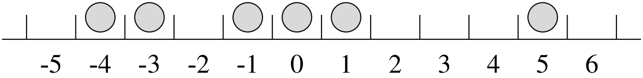
- $p(x,y)$ – underlying random walk
- $g\left(\tau(N_{x,y})\right)$ – reaction with the neighbourhood $N_{x,y}$



IPS suitable for traffic modelling



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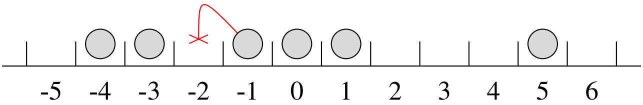

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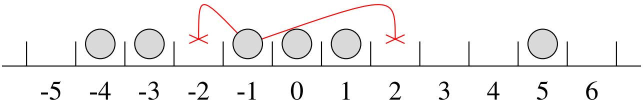
- particles cannot overtake (*discutable*)



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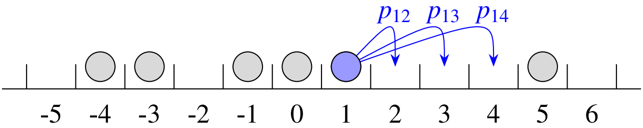
$$g\big(\tau(N_{x,y})\big) = 0 \Leftarrow \begin{cases} y < x & \text{backward movement} \\ \exists\, z\,(x < z < y)(\tau_z = 1) & \text{overtaking} \end{cases}$$



IPS suitable for traffic modelling

- particles cannot move backwards
- particles cannot overtake (*discutable*)
- the range of $N_{x,y}$ is “conditionally” restrained (*for simplicity*)

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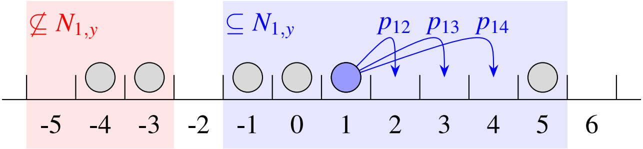

Stationary distribution $\mathcal{P}$

- state space $S = \{0,1\}^{\mathbb{L}}$, $\mathbb{L} \subseteq \mathbb{Z}$ is a lattice
- set function $\mathcal{P} : \left(\mathcal{F} \subseteq 2^S\right) \to [0,1] : A \mapsto \mathcal{P}\{\boldsymbol{\tau} \in A\}$
- $\mathbb{L}$ finite $\Rightarrow \mathcal{P}(A) = \sum_{\boldsymbol{\tau} \in A} \mathcal{P}(\boldsymbol{\tau})$

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$A \subseteq S$	$\cdots$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8	$\cdots$
$\boldsymbol{\tau}_a$	$\cdots$	1	0	1	0	0	0	1	1	0	$\cdots$
$\boldsymbol{\tau}_b$	$\cdots$	1	1	1	0	0	0	1	0	0	$\cdots$
$\boldsymbol{\tau}_c$	$\cdots$	1	1	1	0	0	0	1	1	0	$\cdots$
$\boldsymbol{\tau}_d$	$\cdots$	0	1	1	0	0	0	1	1	0	$\cdots$
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$$A = \{\boldsymbol{\tau} \mid (\tau_2, \ldots, \tau_6) = (10001)\}$$

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- Kolmogorov consistency conditions

$$\mathcal{P}_n(s_1, \dots, s_n) = \sum_s \mathcal{P}_{n+1}(s_1, \dots, s_n, s) = \sum_s \mathcal{P}_{n+1}(s, s_1, \dots, s_n)$$

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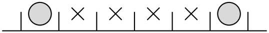
- density $\varrho \in [0,1]$ = average occupation of the site

$$\varrho = \mathcal{P}_1(1)\,, \quad \sigma := 1-\varrho = \mathcal{P}_1(0)$$

Distance-headway and block-length distribution

- distance-headway probability $n \geq 0$



$$P^{dh}(n) = \mathcal{P}\left(\underline{1}\,\overbrace{00\ldots0}^{n}\,1 \mid \tau_0 = 1\right) = \frac{\mathcal{P}_{n+2}\left(1\,\overbrace{00\ldots0}^{n}\,1\right)}{\mathcal{P}_1(1)}$$

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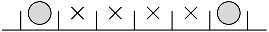

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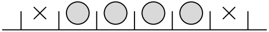
- block-length probability $m \geq 0$



$$Q^{bl}(m) = \mathcal{P}\big(\underline{0}\,\overbrace{11\ldots1}^{m}\,0 \mid \tau_0 = 0\big) = \frac{\mathcal{P}_{n+2}\big(0\,\overbrace{11\ldots1}^{m}\,0\big)}{\mathcal{P}_1(0)}$$

Probability measure Pr on the trajectory space

- Markov process $\big(\boldsymbol{\tau}(t), t \in T\big)$, where $\boldsymbol{\tau}(t) \sim \mathcal{P}$ (stationary distribution)
- trajectory $\boldsymbol{\tau}(\cdot) \in S^T$, sigma-field $\mathcal{G} \subseteq S^T$
- set function $\Pr : \mathcal{G} \to [0,1] : B \mapsto \Pr\big[\boldsymbol{\tau}(\cdot) \in B\big]$

$\boldsymbol{\tau}(\cdot) \subseteq S^{\mathbb{Z}}$	$\cdots$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8	$\cdots$
$t = -1$	$\cdots$	0	1	0	0	1	0	1	0	0	$\cdots$
$t = 0$	$\cdots$	0	1	0	0	0	1	1	0	0	$\cdots$
$t = 1$	$\cdots$	0	0	1	0	0	1	0	1	0	$\cdots$
$t = 2$	$\cdots$	1	0	0	1	0	0	1	0	1	$\cdots$
$t = 3$	$\cdots$	1	0	0	0	1	0	1	0	0	$\cdots$
$t = 4$	$\cdots$	0	1	0	0	0	1	0	1	0	$\cdots$

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$t=0$	$\cdots$	0	1	0	0	0	1	1	0	0	$\cdots$
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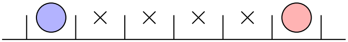
$t=2$	$\cdots$	1	0	0	1	0	0	1	0	1	$\cdots$
$t=3$	$\cdots$	1	0	0	0	1	0	1	0	0	$\cdots$
$t=4$	$\cdots$	0	1	0	0	0	1	0	1	0	$\cdots$

$$B = \big\{\boldsymbol{\tau}(\cdot) \mid \tau_4\big([-1,0,1,2,3,4]\big) = (1,0,0,0,1,0)\big\}$$

Step-headway distribution

- consider discrete time $T = \mathbb{Z}$
- leading particle ○, following particle ○



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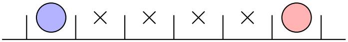

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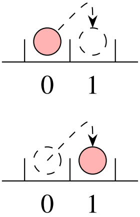

- LP hops from site 0 to 1 at $t = 0$

Step-headway distribution

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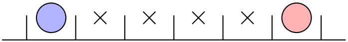


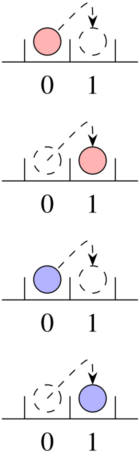
- LP hops from site 0 to 1 at $t = 0$
- FP hops from site 0 to 1 at $t = t_{\text{out}}$

$t = -1$

$t = 0$

$t = t_{\text{out}} - 1$

$t = t_{\text{out}}$



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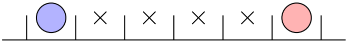
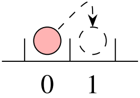


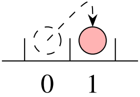
- LP hops from site 0 to 1 at $t = 0$
- FP hops from site 0 to 1 at $t = t_{\text{out}}$
- step-headway probability

$$f(k) = \Pr\left(t_{\text{out}} = k \mid \text{LP } 0 \to 1 \text{ at } t = 0\right)$$

$t = -1$



$t = 0$



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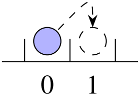
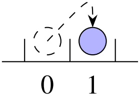


$t = t_{\text{out}}$



Step-headway in detail

t	$\cdots$	-4	-3	-2	-1	0	1	2	3	4	$\cdots$
-1	$\cdots$	0	1	0	0	1	0	1	0	0	$\cdots$
0	$\cdots$	0	1	0	0	0	1	1	0	0	$\cdots$
1	$\cdots$	0	0	1	0	0	1	0	1	0	$\cdots$
2	$\cdots$	1	0	0	1	0	0	1	0	0	$\cdots$
3	$\cdots$	1	0	0	0	1	0	1	0	0	$\cdots$
4	$\cdots$	1	0	0	0	1	0	0	1	0	$\cdots$
5	$\cdots$	1	0	0	0	1	0	0	0	1	$\cdots$
6	$\cdots$	0	1	0	0	0	1	0	0	1	$\cdots$

Step-headway in detail

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-1	$\cdots$	0	1	0	0	1	0	1	0	0	$\cdots$	
0	$\cdots$	0	1	0	0	0	1	1	0	0	$\cdots$	
1	$\cdots$	0	0	1	0	0	1	0	1	0	$\cdots$	
2	$\cdots$	1	0	0	1	0	0	1	0	0	$\cdots$	k_1
3	$\cdots$	1	0	0	0	1	0	1	0	0	$\cdots$	
4	$\cdots$	1	0	0	0	1	0	0	1	0	$\cdots$	
5	$\cdots$	1	0	0	0	1	0	0	0	1	$\cdots$	
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- FP enters 0 after k_1 steps with probability $g(k_1)$

Step-headway in detail

t	$\cdots$	-4	-3	-2	-1	0	1	2	3	4	$\cdots$	
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5	$\cdots$	1	0	0	0	1	0	0	0	1	$\cdots$	$k - k_1$
6	$\cdots$	0	1	0	0	0	1	0	0	1	$\cdots$	

- FP enters 0 after k_1 steps with probability $g(k_1)$
- FP hops from 0 to 1 after $k - k_1$ steps with probability $h(k - k_1 \mid k_1)$

Step-headway in detail

t	$\cdots$	-4	-3	-2	-1	0	1	2	3	4	$\cdots$	
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0	$\cdots$	0	1	0	0	0	1	1	0	0	$\cdots$	
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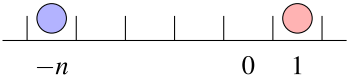
- FP enters 0 after k_1 steps with probability $g(k_1)$
- FP hops from 0 to 1 after $k - k_1$ steps with probability $h(k - k_1 \mid k_1)$

$$f(k) = \sum_{k_1} g(k_1) \cdot h(\, k - k_1 \mid k_1)$$

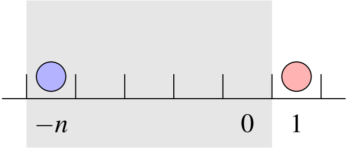
Step-headway in detail: $g(k_1)$

- consider FP n sites behind LP at $t = 0$



Step-headway in detail: $g(k_1)$

- consider FP n sites behind LP at $t = 0$
- "What is probability of that?"



$$P(n) := \Pr\left(\text{FP in } -n \text{ at } t=0 \mid \text{LP left } 0 \text{ at } t=0\right)$$

Step-headway in detail: $g(k_1)$

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- "What is probability of that?"
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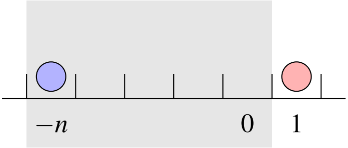

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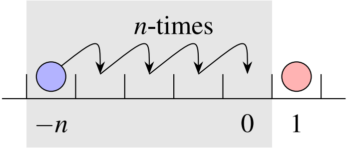

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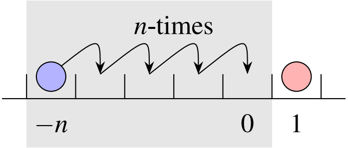

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$$g(k_1) = \sum_{n=1}^{+\infty} P(n) \cdot \Pr\left(k_1 \text{ steps} \mid \text{FP in } -n \text{ at } t=0 \wedge \text{LP...}\right)$$

Step-headway in detail: $h(k - k_1 \mid k_1)$

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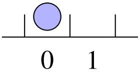

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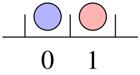

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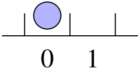

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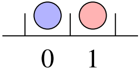

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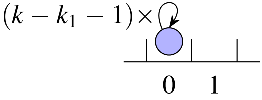

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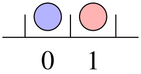

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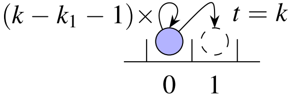

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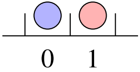

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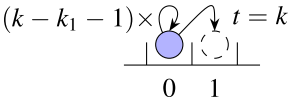

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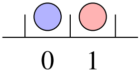

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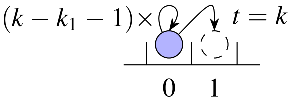

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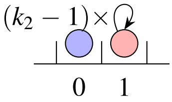

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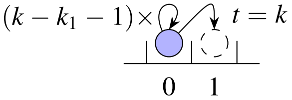

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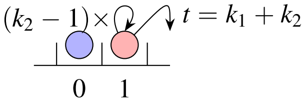

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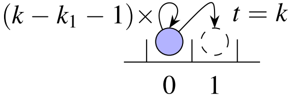

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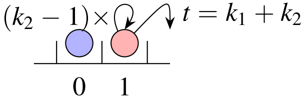

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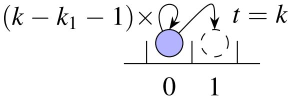

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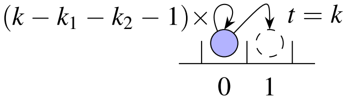

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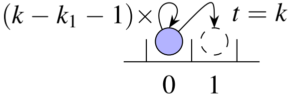

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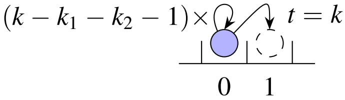

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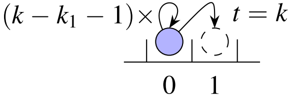

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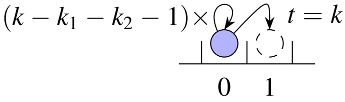

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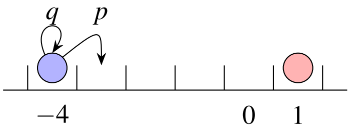
} $\tau_1, \tau_2, \ldots$ in stationary state $\forall\ t$

$$h(k - k_1 \mid k_1) = v(k_1)w(k - k_1) + [1 - v(k_1)] \cdot \sum_{k_2=0}^{k-k_1} u(k_2)w(k - k_1 - k_2)$$

TASEP with forward update

- sites are updated in forward order $\ldots, -1, 0, 1, \ldots$
- site occupied and neighbouring site empty $\implies$ particle hops with probability p or stays with probability $q = 1 - p$
- particle can hop up to the preceding particle



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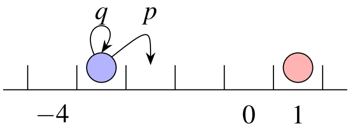

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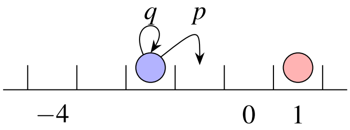

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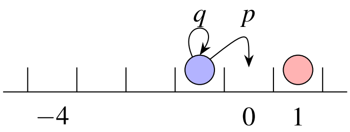

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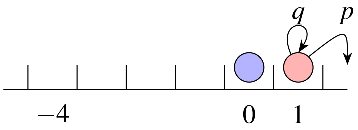

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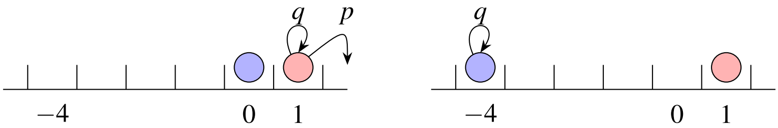

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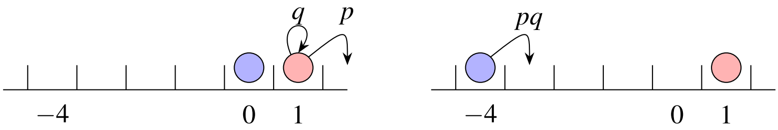

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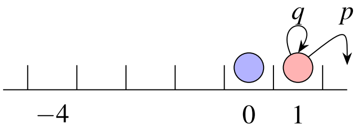

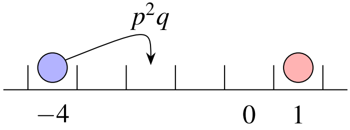

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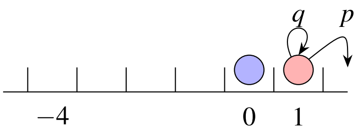

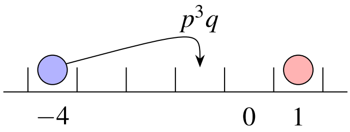

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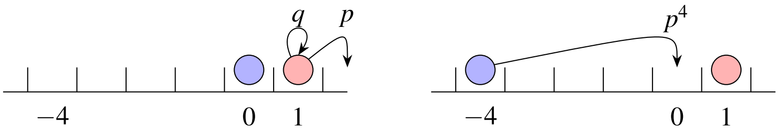

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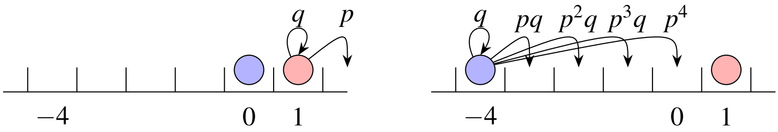
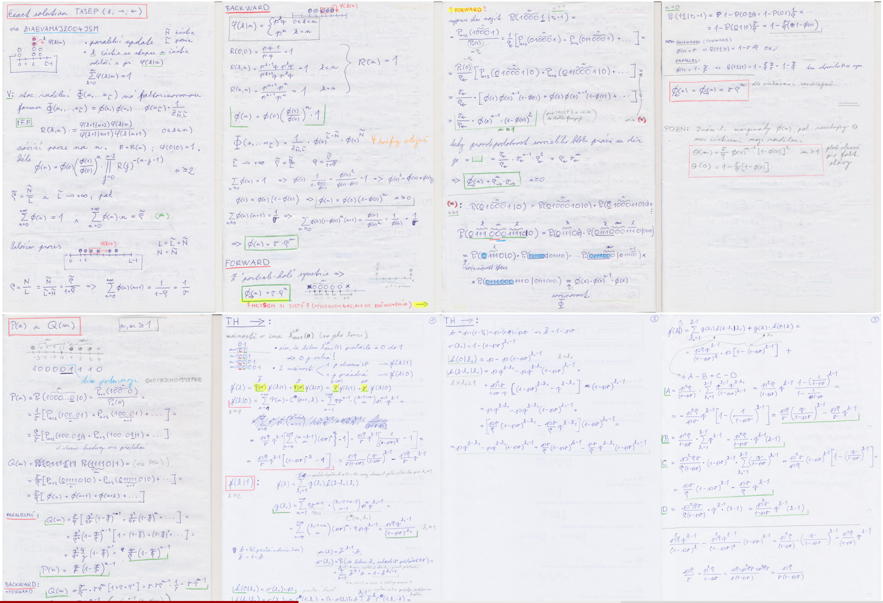


$$p(x, x+n) = p^n q\,, \quad g(N_{x,x+n}) = \begin{cases} 1 & \tau_{x+j} = 0\,,\ j \leq n \wedge \tau_{x+n+1} = 0\,, \\ 1/q & \tau_{x+j} = 0\,,\ j \leq n \wedge \tau_{x+n+1} = 1\,, \\ 0 & \text{otherwise} \end{cases}$$

Some technical but not boring computations



Results to be send to *J. Phys. A*

- distance headway distribution

$$\mathcal{P}_n(\tau_1, \ldots, \tau_n) = \varrho^{\sum \tau_j} \sigma^{n - \sum \tau_j} \implies P^{dh}(n) = \varrho \sigma^n$$

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$$\boxed{f_{\rightarrow}(k) = -p^2 k q^{k-1} + \frac{p\sigma}{\varrho} \left[(1 - p\sigma)^k - q^k \right] + \frac{p\varrho}{q\sigma} \left[\left(\frac{q}{1 - p\sigma} \right)^k - q^k \right]}$$

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- time-headway distribution with $p \rightarrow 0+$ and $t = p \cdot k$

$$\boxed{f(t) = \frac{\varrho}{\sigma}\left(\mathrm{e}^{-\varrho t} - \mathrm{e}^{-t}\right) + \frac{\sigma}{\varrho}\left(\mathrm{e}^{-\sigma t} - \mathrm{e}^{-t}\right) - t\mathrm{e}^{-t}}$$

Comparison with the real-traffic data

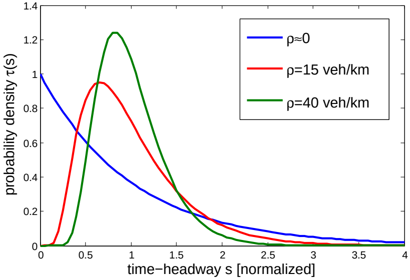
- normalization is necessary : $t \to s$, $\langle \Delta s \rangle = 1$

$$\tau(s) = \langle \Delta t \rangle f(t)\,, \qquad s = t/\langle \Delta t \rangle\,, \qquad \langle \Delta t \rangle = 1/\varrho\sigma\,.$$

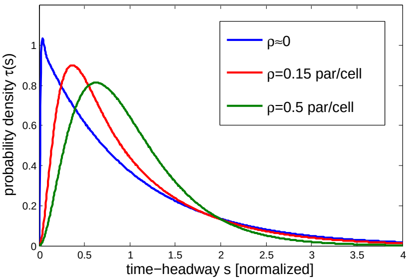
$$\tau(s) = \frac{1}{\sigma^2} e^{-s/\sigma} + \frac{1}{\varrho^2} e^{-s/\varrho} - \left(\frac{1}{\sigma^2} + \frac{1}{\varrho^2}\right) e^{-s/\sigma\varrho}\,.$$

- "particle-hole" symmetry $\varrho \leftrightarrow 1 - \varrho$

real traffic



TASEP



Thank you for your attention!