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# Dynamic Model Averaging for Combining Inferences from Competing Statistical Models

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## 1 Dynamic Model Averaging for Continuous Outcomes

Bayesian model averaging (BMA) is the principled Bayesian approach to combining inferences from competing statistical models. We extend BMA to the dynamic situation where predictions are updated as observations arrive and there is model uncertainty. In Dynamic Model Averaging (DMA) a state space model for the parameters of each model is combined with a Markov chain model for the correct model, which changes over time. The state space and Markov chain models are both specified in terms of forgetting, giving parsimony. For continuous outcomes, our model combines an observation equation and a state equation, both of which are specified for each of the competing models. Typically the observation equation takes the form of a linear regression with normal errors. The resulting method was introduced by [4]. The model has several important special cases:

- When there is only one model, DMA reduces to standard Kalman filtering.
- When the model and parameters do not change over time, so that the problem is the standard one of inference for an unknown but constant model in the presence of model uncertainty, DMA is a recursive version of BMA, called Recursive Model Averaging (RMA). This is a computationally efficient implementation of BMA, which is of interest because standard (non-recursive) implementations of BMA can be computationally demanding.
- When the parameters change over time but the model does not, DMA becomes inference for a time-varying parameter regression model in the presence of model uncertainty.

We will describe an application to online prediction of the output of a cold rolling mill, intended as part of an adaptive control system. DMA has been successfully applied in economics, to forecasting inflation and other macroeconomic quantities [1, 2]. It performed considerably better than a wide range of standard alternative econometric approaches.

## 2 Dynamic Model Averaging for Binary Outcomes

We extend the method to binary outcomes by changing the observation equation from a normal linear regression model to a logistic regression model. The state equation is unchanged. The resulting updating method uses the Laplace method to approximate the posterior distribution of the state given data up to the present [3]. We apply our method to data from children with appendicitis who receive either a traditional (open) appendectomy or a laparoscopic procedure. Our results indicate that predictors of the type of procedure changed significantly over the seven years of data collection, a feature which is not captured using standard logistic regression modeling. Because our procedure

can be implemented completely online, future data collection for similar studies would only require storing sensitive patient information temporarily, reducing the risk of a breach of confidentiality.

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