Sharing of Knowledge and Preferences among Imperfect Bayesian Participants

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Decision Making (DM) with Limited Resources

Closed DM loop

Participant
- limited resources
- domain-specific

Interface
- ⇆ knowledge
- → actions

Environment
- uncertainty
- changes

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- $\leftrightarrows$ knowledge
- $\longrightarrow$ actions

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Prescriptive Bayesian decision-making theory treats DM elements, i.e., participant's preferences, constraints and knowledge consisting of

- observation of the environment's response on actions
- previously accumulated prior knowledge.

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How to

- map domain-specific DM elements on Bayesian ones?
- share participant's DM elements with others in its environment?

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Limited evaluation resources of the participant have to be respected!

Formalisation of Decision Making

- A participant selects and uses
  strategy $\mathsf{s} \in \mathbf{s} \equiv \{\mathsf{s} : \mathbf{k} \to a \in \mathbf{a}\}$ to reach a preferred
  behaviour $b = [g, a, k]$ =[ignorance, action, knowledge] with
  ignorance consisting of unobserved and unknown variables.

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- DM under uncertainty arises unless a participant can uniquely assign b to s. Then performance index I_s cannot be optimised and the optimal strategy $^{\mathsf{I}}\mathsf{s}$ is a minimiser of its expectation $\mathsf{E}_\mathsf{s}[\mathsf{I}_\mathsf{s}]$

$$^{\mathsf{I}}\mathsf{s} \in \mathrm{Arg}\min_{\mathsf{s}\in\mathbf{s}} \mathsf{E}_\mathsf{s}[\mathsf{I}_\mathsf{s}] = \int_{\mathbf{b}} \mathsf{I}_\mathsf{s}(b)\, \mathsf{f}_\mathsf{s}(b)\, \mathrm{d}b, \tag{1}$$

where f_s is a Radon-Nikodým derivative (rdn) with respect to a strategy-independent product measure $\mathrm{d}b$.

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  where f_s is a Radon-Nikodým derivative (rdn) with respect to a strategy-independent product measure $\mathrm{d}b$.

- f_s describes closed-loop model of the participant and its environment, and it holds $\mathsf{f}_\mathsf{s}(b) = \mathsf{m}(b) \times \mathsf{s}(b)$ = environment model × strategy.

Fully Probabilistic Design (FPD) of Strategy

Optimal strategy ${}^{I}\mathsf{s}$ implied by the chosen I_{s} yields ideal closed-loop model ${}^{I}\mathsf{f}(b) = \mathsf{m}(b) \times {}^{I}\mathsf{s}(b) =$ environment model $\times$ optimal strategy. Thus,

- ${}^{I}\mathsf{f}$ can be used instead of I_{s} to describe the preferred behaviour
- absolutely optimal strategy makes closed-loop model f_{s} equal to ${}^{I}\mathsf{f}$
- a strategy providing f_{s} close to ${}^{I}\mathsf{f}$ can be taken as the optimal one
- Kullback-Leibler divergence (KLD) measures closeness of f_{s} to ${}^{I}\mathsf{f}$.

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FPD selects the strategy minimising KLD of f_{s} on ${}^{I}\mathsf{f}$

$$ {}^{O}\mathsf{s} \in \mathrm{Arg}\min_{\mathsf{s}\in\mathbf{s}} \mathsf{D}(\mathsf{f}_{\mathsf{s}}||\,{}^{I}\mathsf{f}) = \int_{\mathbf{b}} \mathsf{f}_{\mathsf{s}}(b) \ln\left(\frac{\mathsf{f}_{\mathsf{s}}(b)}{{}^{I}\mathsf{f}(b)}\right) \mathrm{d}b \qquad (2) $$

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Important features

FPDs densely extend the standard Bayesian designs.

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Important features

FPDs densely extend the standard Bayesian designs.

FPD describes knowledge, constraints & preferences by single language!

FPD: from Math to Real Use

FPD operates on rnds, but participant can provide only domain-specific preferences, constraints and knowledge. How to support their conversion?

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DM elements to be specified

(1) sets of variables forming ignorance **g**, action **a** and knowledge **k**
(2) the set of strategies **s** among which the optimal one ${}^{O}\mathsf{s}(b)$ is searched
(3) the environment model $\mathsf{m}(b)$
(4) the ideal closed-loop model ${}^{\mathsf{I}}\mathsf{f}(b) = {}^{\mathsf{I}}\mathsf{m}(b) \times {}^{\mathsf{I}}\mathsf{s}(b)$.

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Evaluations soon reach complexity boundaries ⇒ distributed solution
How to support distributed DM within FPD?

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Construction of (1) and (2) is supported by hypotheses testing.

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Evaluations soon reach complexity boundaries $\Rightarrow$ distributed solution
How to support distributed DM within FPD?

Construction of (1) and (2) is supported by hypotheses testing.
Construction of (3) and (4) as well as support of distributed DM are addressed by solving appropriate supporting DM tasks.

Supporting DM Tasks

DM elements construction
$\{$domain knowledge, preferences$\} \rightarrow$ rnd

- enriching of knowledge on rnd
- approximation of known rnd

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$\{\text{domain knowledge, preferences}\} \rightarrow \text{rnd}$

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- merging of several rnds
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The supporting DM tasks are formulated and solved via FPD.

Supporting DM Tasks: Notations

- DM elements of supporting DM tasks are denoted by capital letters while that of supported DM task by lower-case letters.

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- A finite cardinality $|\mathbf{b}|$ of the behaviour set $\mathbf{b} = \{b_1, \ldots, b_{|\mathbf{b}|}\}$ of the supported DM is assumed. This implies the inspected rnds f be finite-dimensional vectors

$$\mathsf{f} \in \mathbf{f} \subset \boldsymbol{\Delta} = \left\{ \mathsf{f}(b) : \ \mathsf{f}(b) \geq 0, \ \int_{b \in \mathbf{b}} \mathsf{f}(b) \, \mathrm{d}b = 1 \right\} \qquad (3)$$

Approximating of a Known Rnd f by $\hat{\mathrm{f}}$

	Behaviour $B = [G, A, K]$ = [ignorance,action,knowledge]		
Part	ignorance b	action $\hat{\mathrm{f}}$	knowledge f

Approximating of a Known Rnd f by $\hat{f}$

	Behaviour $B = [G, A, K]$ = [ignorance,action,knowledge]		
Part	ignorance b	action $\hat{f}$	knowledge f
Meaning	original behaviour	approximating rnd	approximated rnd

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Closed-loop model $F(B) = F(b, \hat{f}, f)$

Factors	$F(b\|\hat{f}, f)$	$F(\hat{f}\|f)$	$F(f)$

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Meaning	model of b	strategy	K's rnd

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	Closed-loop model $\mathsf{F}(B) = \mathsf{F}(b, \hat{\mathsf{f}}, \mathsf{f})$		
Factors	$\mathsf{F}(b\|\hat{\mathsf{f}}, \mathsf{f})$	$\mathsf{F}(\hat{\mathsf{f}}\|\mathsf{f})$	$\mathsf{F}(\mathsf{f})$
Choice	$\mathsf{f}(b)$	$\mathsf{S}(\hat{\mathsf{f}}\|\mathsf{f})$	$\mathsf{F}(\mathsf{f})$
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	Ideal closed-loop model ${}^{\mathsf{I}}\mathsf{F}(B)$		
Factors	${}^{\mathsf{I}}\mathsf{F}(b\|\hat{\mathsf{f}}, \mathsf{f})$	${}^{\mathsf{I}}\mathsf{F}(\hat{\mathsf{f}}\|\mathsf{f})$	${}^{\mathsf{I}}\mathsf{F}(\mathsf{f})$

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Factors	$F(b \vert \hat{f}, f)$	$F(\hat{f} \vert f)$	$F(f)$
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Ideal closed-loop model $^{I}F(B)$

Factors	$^{I}F(b \vert \hat{f}, f)$	$^{I}F(\hat{f} \vert f)$	$^{I}F(f)$
Choice	$\hat{f}(b)$	$S(\hat{f} \vert f)$	$F(f)$
Meaning	wish to model b	Left To the Fate	LTF

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	Ideal closed-loop model ${}^{I}F(B)$		
Factors	${}^{I}F(b\|\hat{f}, f)$	${}^{I}F(\hat{f}\|f)$	${}^{I}F(f)$
Choice	$\hat{f}(b)$	$S(\hat{f}\|f)$	$F(f)$
Meaning	wish to model b	Left To the Fate	LTF

- FPD provides the deterministic strategy generating the approximating

$$^{O}\hat{f} \in \operatorname{Arg}\min_{\hat{f} \in \hat{\mathbf{f}}} D(f||\hat{f})$$

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	Ideal closed-loop model ${}^{I}F(B)$		
Factors	${}^{I}F(b\vert\hat{f}, f)$	${}^{I}F(\hat{f}\vert f)$	${}^{I}F(f)$
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- FPD provides the deterministic strategy generating the approximating

$${}^{O}\hat{f} \in \operatorname{Arg}\min_{\hat{f} \in \hat{\mathbf{f}}} D(f\|\hat{f}) \ldots \quad \textit{information criterion recovered}$$

Enriching of Guess f_0 of Unknown f by Domain Knowledge

Behaviour $B = [G, A, K] = [(b, \mathsf{f}), \mathsf{F}(\mathsf{f}|K), K]$

Part	ignorance $b, \mathsf{f}(b)$	action $A = \mathsf{F}(\mathsf{f}|K)$	knowledge $K = \mathbf{f}$
Meaning	behaviour, its rnd	description of f	set of fs

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Choice	$\mathsf{f}(b)$	A	$\mathsf{S}(A|K)$	$\mathsf{F}(K)$
Meaning	model of b	action	strategy	K's rnd

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	Closed-loop model $F(B) = F(b, f, A, K)$			
Factors	$F(b|f, A, K)$	$F(f|A, K)$	$F(A|K)$	$F(K)$
Choice	$f(b)$	A	$S(A|K)$	$F(K)$
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	Ideal closed-loop model ${}^{I}F(B)$			
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Behaviour $B = [G, A, K] = [(b, \mathsf{f}), \mathsf{F}(\mathsf{f}|K), K]$

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Ideal closed-loop model ${}^{\mathsf{I}}\mathsf{F}(B)$

Factors	${}^{\mathsf{I}}\mathsf{F}(b|\mathsf{f}, A, K)$	${}^{\mathsf{I}}\mathsf{F}(\mathsf{f}|A, K)$	${}^{\mathsf{I}}\mathsf{F}(A|K)$	${}^{\mathsf{I}}\mathsf{F}(K)$
Choice	$\mathsf{f}_0(b)$	A	$\mathsf{S}(A|K)$	$\mathsf{F}(K)$
Meaning	prior model of b	Left To the Fate	LTF	LTF

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Part	ignorance $b, f(b)$	action $A = F(f\|K)$	knowledge $K = \mathbf{f}$
Meaning	behaviour, its rnd	description of f	set of fs

	Closed-loop model $F(B) = F(b, f, A, K)$			
Factors	$F(b\|f, A, K)$	$F(f\|A, K)$	$F(A\|K)$	$F(K)$
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	Ideal closed-loop model ${}^{I}F(B)$			
Factors	${}^{I}F(b\|f, A, K)$	${}^{I}F(f\|A, K)$	${}^{I}F(A\|K)$	${}^{I}F(K)$
Choice	$f_0(b)$	A	$S(A\|K)$	$F(K)$
Meaning	prior model of b	Left To the Fate	LTF	LTF

- FPD provides the deterministic strategy with the deterministic action

$$ {}^{O}f \in \mathrm{Arg}\min_{f \in \mathbf{f}} D(f||f_0) $$

Enriching of Guess f_0 of Unknown f by Domain Knowledge

	Behaviour $B = [G, A, K] = [(b, \mathsf{f}), \mathsf{F}(\mathsf{f}\vert K), K]$			
Part	ignorance $b, \mathsf{f}(b)$	action $A = \mathsf{F}(\mathsf{f}\vert K)$	knowledge $K = \mathbf{f}$	
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	Ideal closed-loop model ${}^{\mathsf{I}}\mathsf{F}(B)$			
Factors	${}^{\mathsf{I}}\mathsf{F}(b\vert\mathsf{f}, A, K)$	${}^{\mathsf{I}}\mathsf{F}(\mathsf{f}\vert A, K)$	${}^{\mathsf{I}}\mathsf{F}(A\vert K)$	${}^{\mathsf{I}}\mathsf{F}(K)$
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$$ {}^{O}\mathsf{f} \in \mathrm{Arg}\min_{\mathsf{f}\in\mathbf{f}} \mathsf{D}(\mathsf{f}||\mathsf{f}_0) \quad \ldots \quad \textit{minimum KLD principle recovered} $$

$$ \Leftrightarrow \quad \textit{maximum entropy principle for uniform } \mathsf{f}_0 $$

Enriching of Prior Rnd F_0 of an Unknown f by Rnds' Set

	Behaviour $B = [G, A, K] = [(b, \mathsf{f}), \mathsf{F}(\mathsf{f}|K), K]$		
Part	ignorance $b, \mathsf{f}(b)$	action $A = \mathsf{F}(\mathsf{f}|K)$	$K = \mathbf{f}$
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	Closed-loop model $F(B) = F(b, f, A|K)F(K)$			
Factors	$F(b|f, A, K)$	$F(f|A, K)$	$F(A|K)$	$F(K)$

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Factors	$\mathsf{F}(b|\mathsf{f}, A, K)$	$\mathsf{F}(\mathsf{f}|A, K)$	$\mathsf{F}(A|K)$	$\mathsf{F}(K)$
Choice	$\mathsf{f}(b)$	A	$\mathsf{S}(A|K)$	$\mathsf{F}(K)$
Meaning	model of b	action	strategy	K's rnd

Enriching of Prior Rnd F_0 of an Unknown f by Rnds' Set

	Behaviour $B = [G, A, K] = [(b, \mathsf{f}), \mathsf{F}(\mathsf{f}|K), K]$		
Part	ignorance $b, \mathsf{f}(b)$	action $A = \mathsf{F}(\mathsf{f}|K)$	$K = \mathbf{f}$
Meaning	behaviour, its rnd	description of f	set of fs

	Closed-loop model $\mathsf{F}(B) = \mathsf{F}(b, \mathsf{f}, A|K)\mathsf{F}(K)$			
Factors	$\mathsf{F}(b|\mathsf{f}, A, K)$	$\mathsf{F}(\mathsf{f}|A, K)$	$\mathsf{F}(A|K)$	$\mathsf{F}(K)$
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	Ideal closed-loop model ${}^{\mathsf{I}}\mathsf{F}(B)$			
Factors	${}^{\mathsf{I}}\mathsf{F}(b|\mathsf{f}, A, K)$	${}^{\mathsf{I}}\mathsf{F}(\mathsf{f}|A, K)$	${}^{\mathsf{I}}\mathsf{F}(A|K)$	${}^{\mathsf{I}}\mathsf{F}(K)$

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Factors	$\mathsf{F}(b\vert\mathsf{f}, A, K)$	$\mathsf{F}(\mathsf{f}\vert A, K)$	$\mathsf{F}(A\vert K)$	$\mathsf{F}(K)$
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Choice	$\mathsf{f}(b)$	$\mathsf{F}_0(\mathsf{f})$	$\mathsf{S}(A\vert K)$	$\mathsf{F}(K)$
Meaning	model of b	prior guess of A	Left To the Fate	LTF

Enriching of Prior Rnd F_0 of an Unknown f by Rnds' Set

	Behaviour $B = [G, A, K] = [(b, f), F(f \mid K), K]$		
Part	ignorance $b, f(b)$	action $A = F(f \mid K)$	$K = \mathbf{f}$
Meaning	behaviour, its rnd	description of f	set of fs

	Closed-loop model $F(B) = F(b, f, A \mid K)F(K)$			
Factors	$F(b \mid f, A, K)$	$F(f \mid A, K)$	$F(A \mid K)$	$F(K)$
Choice	$f(b)$	A	$S(A \mid K)$	$F(K)$
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Factors	${}^{I}F(b \mid f, A, K)$	${}^{I}F(f \mid A, K)$	${}^{I}F(A \mid K)$	${}^{I}F(K)$
Choice	$f(b)$	$F_0(f)$	$S(A \mid K)$	$F(K)$
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- FPD generalises the minimum KLD principle

$$
{}^{O}F(f \mid K) \in \operatorname{Arg}\min_{F \in \mathbf{F}} \int_{\mathbf{f}} F(f) \ln \left(\frac{F(f)}{F_0(f)} \right) \mathrm{d}f
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Supporting DM Tasks in Service of Supported DM Tasks

DM elements construction
$\{$domain knowledge, preferences$\} \rightarrow$ rnd

- enriching of knowledge on rnd
- approximation of known rnd

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$$\mathsf{E}_{\mathsf{f}}[\phi_\kappa] = \int_{\mathbf{b}} \phi_\kappa(b)\mathsf{f}(b)\,\mathrm{d}b = 0,\ \kappa \in \boldsymbol{\kappa} = \{1, 2, \ldots, |\boldsymbol{\kappa}|\},\ |\boldsymbol{\kappa}| < \infty, \quad (\star).$$

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Note: The construction was used for knowledge elicitation. Similarly it can be applied to the ideal rnd construction, i.e., to the preference elicitation.

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Distributed DM: Merging of Probabilistic Knowledge

- Let set $\mathbf{f}$ contain possible compromises between several given rnds $\{\mathsf{f}_\kappa(b)\}_{\kappa\in\boldsymbol{\kappa}} \in \boldsymbol{\Delta}$ and $\mathsf{F}_0(\mathsf{f})$ be the prior rnd on $\mathbf{f}$.
- The KLD $\mathsf{D}(\mathsf{f}_\kappa||\mathsf{f})$ specifies how well $\mathsf{f} \in \mathbf{f}$ approximates f_κ, i.e., the acceptability of f as the compromise for the κ-th participant. Thus, given thresholds $\{\beta_\kappa\}_{\kappa\in\boldsymbol{\kappa}}$ specify the meaningful knowledge on f

$$K: \; \mathsf{E}_\mathsf{F}[\mathsf{D}(\mathsf{f}_\kappa||\mathsf{f})] \le \beta_\kappa \in (0,\infty), \;\; \kappa \in \boldsymbol{\kappa}. \tag{4}$$

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gives ${}^{O}\mathsf{F}(\mathsf{f}) = \mathcal{D}\left[\nu_0 + \sum_{\kappa\in\boldsymbol{\kappa}} \lambda_\kappa \mathsf{f}_\kappa\right]$, where $\lambda_\kappa \ge 0$ are Kuhn-Tucker multipliers and $\hat{\mathsf{f}}(b) = \mathsf{E}[\mathsf{f}(b)|K]$ is affine combination of $\{\mathsf{f}_\kappa\}_{\kappa\in\boldsymbol{\kappa}}$.

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Merging allows imperfect participants cooperate by sharing “personal” rnds.

Extension of Fragmental Knowledge for Merging

- Imperfect participant provides fragmental knowledge or preferences leading to a rnd $\mathsf{f}(m_\kappa|k_\kappa)$ derived from $\mathsf{f}_\kappa(b)$ with

$$\begin{aligned} b &= [u_\kappa, m_\kappa, k_\kappa] = \text{behaviour split to parts} \qquad (5)\\ &= [\text{uninteresting for, modelled by, known to}]\ \ \kappa\text{th rnd provider} \end{aligned}$$

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- The construction of the merger $\hat{\mathsf{f}}(b)$ is ready if the respective rnds $\mathsf{f}_\kappa(m_\kappa|k_\kappa)$ are extended to the full behaviour b.
- The extension should be the best approximation of the merger. This together with the merging formula gives

$$\hat{\mathsf{f}}(b) = \frac{\nu_0(b) + \sum_{\kappa\in\boldsymbol{\kappa}} \lambda_k \hat{\mathsf{f}}(u_\kappa|m_\kappa, k_\kappa)\mathsf{f}_\kappa(m_\kappa|k_\kappa)\hat{\mathsf{f}}(k_\kappa)}{\int_{\mathbf{b}} \nu_0(b)\,\mathrm{d}b + \sum_{\kappa\in\boldsymbol{\kappa}} \lambda_\kappa}.$$

- The merger is projected back to $(\mathbf{m}_\kappa, \mathbf{k}_\kappa)$ of cooperating imperfect participants. This corrects DM elements understandable to them.

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- Can we
  - learn from nature/society something radically different from the approach?
  - use the approach for modelling natural/societal systems?