# Data dropouts in Bayesian model averaging for application in cold rolling

(experiments with potential methods)

### Ladislav Jirsa

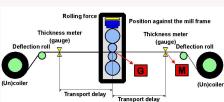
ÚTIA AV ČR, v.v.i. Prague, Czech Republic

6. 12. 2010. ÚTIA. seminar AS



# Cold metal rolling





### Problems and tasks

To control the rolling plant, the controlled quantity (output thickness of the metal strip) must be known.

### **Problems**

- metal thickness cannot be measured in the rolling gap
- output metal thickness is measured with a high transport delay (19, 123)
- faulty and noisy data

### Tasks

- predict the non-delayed output thickness ("soft sensor")
- use mixture of multiple models with dynamic weights



### The idea of BMA

We use K models  $M^1, M^2, ..., M^K$  for prediction of the target quantity p

$$\operatorname{data} \to \left\{ \begin{array}{cccc} \operatorname{model} M^1 & \to & \operatorname{predictor} p | M^1 & \to & \operatorname{weight} \omega^1 \\ \operatorname{model} M^2 & \to & \operatorname{predictor} p | M^2 & \to & \operatorname{weight} \omega^2 \\ \dots & \dots & \dots & \dots \\ \operatorname{model} M^K & \to & \operatorname{predictor} p | M^K & \to & \operatorname{weight} \omega^K \end{array} \right\} \to p = \sum_{i=1}^K \omega^i p | M^i$$

Dynamic model averaging — allows time changes of parameters and weights



# Estimation and prediction

data 
$$d_t = [y_t, u'_t]'$$
 parameter  $\Theta_t$ 

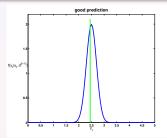
- single model M
  - estimation of slowly varying parameters
    - time update: forgetting (exponential, stabilized, partial, not linear), parameter  $\lambda$
    - data update: Bayes formula

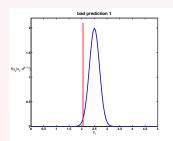
• prediction: 
$$f(y_t|u_t, d^{(t-1)}, M) = \int f(y_t|u_t, d^{(t-1)}, \Theta_t, M) f(\Theta_t|d^{(t)}, M) d\Theta_t$$

- multiple models  $M^i$ , i = 1, ..., K
  - estimation and prediction for each model
  - ullet estimation of varying weights  $\omega^i_{t|t}$  based on predictions
    - time update: linear forgetting,  $\omega_{t-1|t-1}^i \to \omega_{t|t-1}^i$ , parameter  $\alpha$
    - data update: Bayes formula  $\omega_{t|t}^i \propto f(y_t|u_t, d^{(t-1)}, M^i) \; \omega_{t|t-1}^i$

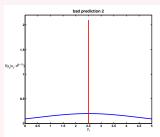


# Quality of prediction





- measured data give high value of predictive density → "good" prediction
- $\bullet$  measured data give low value of predictive density  $\rightarrow$  "bad" prediction





# Linear regression model with normal noise

- model  $y_t = \vartheta_t' \psi_t + e_t$  noise  $e_t \sim \mathcal{N}(0, r_t)$  parameter  $\Theta_t = [\vartheta_t', r_t]'$
- regressor  $\psi_t \subset \{u_t, y_{t-1}, u_{t-1}, ..., y_{t-\partial}, u_{t-\partial}\}$  data vector  $\phi_t = \left[y_t, \psi_t'\right]'$
- expressing belief in data: data weights  $w_t \in \langle 0, 1 \rangle$
- conjugate system Gauss-inverse-Wishart

$$\begin{array}{cccc} f\left(\Theta_{t}|\textit{d}^{(t)}\right) & \equiv & f\left(\Theta_{t}|\textit{V}_{t},\nu_{t}\right) \\ \textit{V}_{t} & = & \lambda\textit{V}_{t-1} + \textit{W}_{t}^{2} & \phi_{t}\phi_{t}' \\ \nu_{t} & = & \lambda\nu_{t-1} + \textit{W}_{t} \end{array} \right\} \text{ exponential forgetting with data weights}$$

- ullet decomposition of extended information matrix  $V_t = L_t' D_t L_t$ 
  - L<sub>t</sub> lower triangular with unit diagonal
  - D<sub>t</sub> diagonal matrix
- estimate of noise variance  $\hat{r}_t = \frac{D_{11,t}}{\nu_t 2}$  predictive variance  $\hat{r}_{p,t+1|t} > \hat{r}_t$

Weights depend on predictive pdf, its variance depends on  $D_{11}$ .



# Dealing with data dropouts

Let be available quantity  $a_{jt} \in \langle 0, 1 \rangle$  expressing reliability of j-th data channel in time t

For  $\phi_t^i$  get  $a_t^i$  as a product of corresponding entries of  $a_{jt}$ 

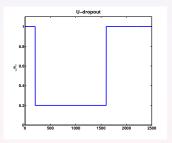
These methods were tested as possible tools:

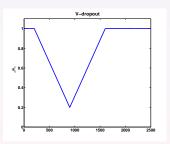
- assigning zero values to the data
- direct modification of D<sub>11,t</sub>
- **1** modification of forgetting factor  $\lambda$
- **4** assigning weights to the data vector  $\phi_t$

Each method affects a different link of the chain:

$$\mathsf{data} \to \left\{ \begin{array}{ccc} \mathsf{model} \ M^1 & \to & \mathsf{predictor} \ \rho | M^1 & \to & \mathsf{weight} \ \omega^1 \\ \mathsf{model} \ M^2 & \to & \mathsf{predictor} \ \rho | M^2 & \to & \mathsf{weight} \ \omega^2 \\ \dots & \dots & \dots \\ \mathsf{model} \ M^K & \to & \mathsf{predictor} \ \rho | M^K & \to & \mathsf{weight} \ \omega^K \\ \end{array} \right\} \to \rho = \sum_{i=1}^K \omega^i \ \rho | M^i$$

# Types of data dropouts used in the experiments





- u-dropout, v-dropout
- $b_1 \leq a_t^i \leq b_2$
- both  $b_1$  and  $b_2$  can be chosen
- in the experiments,  $b_2 = 1$
- here,  $b_1 = 0.2$



# Models used for mixing

These models were used for experiments:

- **1** derived from mass-flow principle,  $\psi = [v_r, h_1 v_r, 1]'$
- ② derived from gaugemeter principle linear force,  $\psi = [z, F, 1]'$
- **3** simplest "gray box" model,  $\psi = [h_1, z, 1]'$
- **1** another "gray box" model,  $\psi = [h_1, z, v_r, 1]'$
- **5** derived from gaugemeter principle quadratic force,  $\psi = [z, F, F^2, 1]'$

# Assigning zero values to the data

### Principle:

- initial attempt for binary  $a_{it} \in \{0, 1\}$
- it leaves the predictive variance to its fate
- based on assumption that zero-valued data channels will decrease prediction ability and hence the weights of the affected model

### Advantages:

natural and implicit mechanism

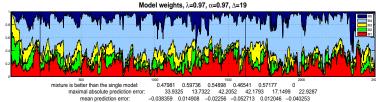
### Disadvantages:

it does not work



# Experiments with the method

#### No dropout



 maximal absolute prediction error:
 33.9325
 13.7322
 42.052
 42.1793
 17.1499
 22.9

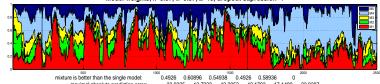
 mean prediction error:
 -0.03859
 0.10480
 -0.0256
 -0.052713
 0.10246
 -0.00257

 median of prediction error:
 0.01897
 0.16452
 0.023774
 -0.044317
 0.092584
 -0.044814

 obstraction of the model:
 0.18562
 0.096605
 0.11604
 0.47951
 0.11463

#### Dropout in h<sub>1</sub> between 200 and 1600 (zeroes), inputs of models 3 and 4 affected

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=1



mixture is better than the single model:
maximal absolute prediction error:
mean prediction error:
—0.0
median of prediction error:
5td. dev. of prediction error:
mean weights of the model:

### Direct modification of $D_{11}$

### Principle:

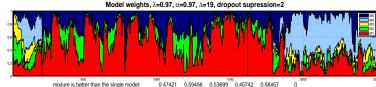
- model weight  $\omega_{t|t}^{i}$  is affected by the predictive variance given by  $D_{11,t}^{i}$
- significant increase of the predictive variance
- $D_{11,t} \leftarrow D_{11,t}^i + (1-a_t^i) C$ ,
- C is chosen to be "much" greater than original  $D_{11,t}^i$  to increase predictive variance

#### Effect:

- immediate suppression of the respective  $\omega_{t|t}^i$  if  $a_t^i < 1$
- if  $a_t^i=$  1, return to the regular operating state is slow and it is given by  $\lambda$  and C

# Experiments with the method I

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0, 1)$ ,  $C = 10^3$ 



mixture is better than the single model:

maximal absolute prediction error:

mean prediction error:

-0.0

median of prediction error:

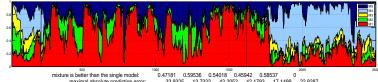
ot. dev. of prediction error:

mean weights of the model:

0.44(21 0.59456 0.50899 0.46472 0.59454 0.4727 0.59454 0.339325 13,7322 42.2052 42.1793 17.1499 22.93 0.03839 0.014908 0.002256 0.0052713 0.012046 0.015525 0.010677 0.10452 0.02374 0.044317 0.092584 0.016707 0.32056 3.5595 3.3059 3.5597 3.3064 0.9452 0.94526 0.04522 0.16172 0.18505

V-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0, 1)$ ,  $C = 10^3$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=2

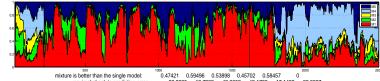


mixture is better than the single model: 0
maximal absolute prediction error:
mean prediction error: -0.03
median of prediction error: 0.0'
std. dev. of prediction error:
mean weights of the model:

# Experiments with the method II

U-dropout in  $h_1$  between 200 and 600,  $(b_1, b_2) = (0.5, 1)$ ,  $C = 10^3$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=2



mixture is better than the single model:

maximal absolute prediction error:

mean prediction error:

olimitation of prediction error:

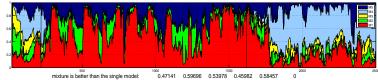
std. dev. of prediction error:

mean weights of the model:

0.35936 0.35936 0.35936 0.35936 0.35936 0.35936 0.35936 0.3256 0.35736 0.02556 0.05273 0.01246 0.01245 0.016877 0.10452 0.023774 0.044317 0.092584 0.01245 0.32574 0.044317 0.092584 0.01245 0.32574 0.044317 0.092584 0.01687 0.35956 0.35956 0.35956 0.35957 0.3029 0.35957 0.3038 0.3595 0.34418 0.042595 0.17014 0.18131

V-dropout in  $h_1$  between 200 and 600,  $(b_1, b_2) = (0.5, 1)$ ,  $C = 10^3$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=2

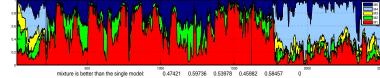


mixture is better than the single model: maximal absolute prediction error: mean prediction error: median of prediction error: std. dev. of prediction error:

# Experiments with the method III

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.5, 1)$ ,  $C = 10^2$ 



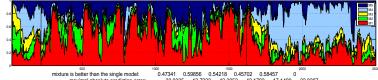


maximal absolute prediction error:
mean prediction error:
-0.038
median of prediction error:
std. dev. of prediction error:
mean weights of the model:

0.41421 0.39736 0.53976 0.79982 0.59976 9 22.9 0.3339325 13.7322 42.2052 42.1793 17.1499 22.9 0.010877 0.10452 0.02276 -0.04237 0.012046 -0.0063400 0.010877 0.10452 0.02374 -0.044317 0.092584 -0.017578 3.2356 3.5536 3.5573 3.309 3.5597 3.301 del: 0.43927 0.14132 0.049548 0.18743 0.17483

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.5, 1)$ ,  $C = 10^1$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=2



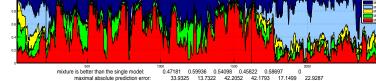
mixture is better than the single model: 0
maximal absolute prediction error:
mean prediction error: -0.03
median of prediction error: 0.0'
std. dev. of prediction error:
mean weights of the model:

0.47.341 0.59956 0.54218 0.45702 0.58457 0 33.9325 13.7322 42.2052 42.1793 17.1499 22.925 0.038359 0.014908 -0.02256 -0.052713 0.012046 -0.0092456 0.010877 0.10452 0.02374 -0.044317 0.092584 -0.01874 0.010877 0.10452 0.02374 -0.044317 0.092584 -0.01874 0.2256 3.5536 3.5977 3.2009 3.5597 3.3047 del: 0.38546 0.13265 0.076847 0.2344 0.16204

# Experiments with the method IV

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.9, 1)$ ,  $C = 10^2$ 



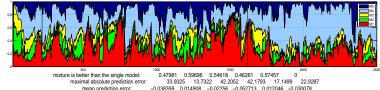


maximal absolute prediction error:
mean prediction error:
-0.03t
median of prediction error:
std. dev. of prediction error:
mean weights of the model:

23.925 13.7322 42.2052 21.793 17.1499 22.9. -0.03839 0.014908 -0.02256 -0.052713 0.012046 -0.005865 0.010877 0.10452 0.023774 -0.044317 0.092584 -0.023079 3.2356 3.5536 3.5577 3.3009 3.5597 3.3038 del: 0.41232 0.13696 0.064293 0.21052 0.16831

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.9, 1)$ ,  $C = 10^1$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=2



### Advantages:

- fast suppression of the model weights
- value of C can be used to adjust sensitivity to a<sub>t</sub><sup>i</sup>
- during the model weights recovery, the affected models gather enough information for parameters and predictions

### Disadvantages:

- slow recovery of the model weights
- direct manipulation with sufficient statistics
- heuristic



# Modification of forgetting factor $\lambda$

### Principle:

- significant decrease of the predictive variance (close to 0), setting mean value of regression coefficients to 0
- data update

• alternative  $V^{Ai}$ ,  $\nu^{Ai}$  form a proper posterior pdf, keep the computations numerically stable and adjust sensitivity of the method

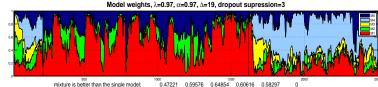
### Effect:

- slower suppression of the respective  $\omega_{t|t}^i$  if  $a_t^i < 1$
- if  $a_t^i = 1$ , return to the regular operating state is very fast



# Experiments with the method I

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0, 1)$ 



kture is better than the single model:

maximal absolute prediction error:

mean prediction error:

output

dev. of prediction error:

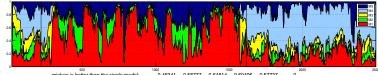
std. dev. of prediction error:

mean weights of the model:

0.47221 0.59576 0.64854 0.60616 0.58297 0 33.9323 13.7319 4.2.052 42.1793 17.0942 22.9 -0.038359 0.014733 0.060504 0.018288 0.01197 -0.01024 0.010888 0.10451 0.10583 0.068812 0.087985 -0.011416 3.2356 3.5531 4.6013 4.5605 3.5586 3.3053 del: 0.42496 0.1391 0.055373 0.20023 0.17274

V-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0, 1)$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=3

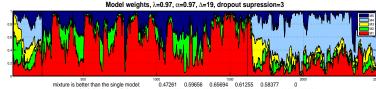


mixture is better than the single model: maximal absolute prediction error: mean prediction error: median of prediction error: std. dev. of prediction error:

ın the single model: 0.46341 0.58737 0.64814 0.60496 0.57737 0 the prediction error: 3.339226 13.7319 79.45426 53.11874 17.09418 22.921 prediction error: 0.008359 0.01473 0.19812 0.13551 0.01197 0.0027304 10.7976diction error: 0.010698 0.10451 -0.08441 -0.12924 0.087985 -0.009377 of prediction error: 3.23564 3.55313 19.8175 14.3834 3.55865 3.34963 mean weights of the model: 0.39794 0.13586 0.067691 0.22433 0.16658

# Experiments with the method II

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.5, 1)$ 

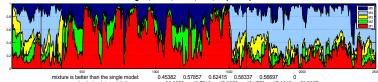


maximal absolute prediction error:
mean prediction error:
-0.038
median of prediction error:
std. dev. of prediction error:
mean weights of the model:

0.47261 0.59656 0.65694 0.61255 0.58377 0 0 33.9323 31,3739 4.22052 42.1793 17.0942 22.9 -0.038359 0.014733 -0.1664 -0.1393 0.01197 -0.0080942 0.010898 0.014951 -0.048443 0.067985 -0.013117 3.2356 3.5586 3.3045 del: 0.42528 0.13919 0.0550995 0.2010 0.17274

V-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.5, 1)$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=3



mixture is better than the single model:

maximal absolute prediction error:

mean prediction error:

o.0.

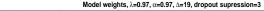
std. dev. of prediction error:

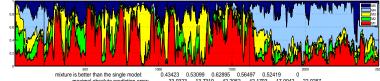
mean weights of the model:

0.45382 0.57857 0.52415 0.58337 0.56691 0 0.333923 1.37919 42.25252 42.1793 17.0942 2.29287 0.39359 0.014733 -0.21036 -0.20424 0.01197 -0.0048862 0.016451 -0.12591 -0.15598 0.067985 0.014699 9.2356 3.5531 4.6114 4.4006 3.5586 3.3708 0 0.36207 0.13252 0.0766646 0.25707 0.16409

# Experiments with the method III

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.9, 1)$ 





ture is better than the single model:

maximal absolute prediction error:

mean prediction error:

-0.0

median of prediction error:

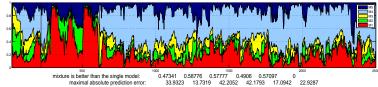
std. dev. of prediction error:

mean weights of the model:

0.43423 0.53099 0.62895 0.56497 0.52419 0 3.93923 13,7319 42.2052 42.1793 17.0942 22.9 -0.038359 0.014733 -0.18761 -0.23454 0.01197 -0.087112 0.010898 0.10451 -0.12556 -0.20644 0.087985 -0.026712 0.087985 -0.20543 0.087985 -0.20543 0.087985 -0.20543 0.087985 -0.20543 0.087985 0.02576 0.087985 0.02576 0.087985 0.087978 0.087985 0.08798

V-dropout in  $h_1$  between 200 and 600,  $(b_1, b_2) = (0.5, 1)$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=3



mixture is better than the single model: 0
maximal absolute prediction error:
mean prediction error: -0.0
median of prediction error: 0.0'
std. dev. of prediction error:
mean weights of the model:

### Advantages:

- faster reaction on  $a_t^i$  in the beginning and the end of the dropout
- statistics V<sup>Ai</sup>, v<sup>Ai</sup> can be used to adjust sensitivity and reaction on status changes; even in the sense of the Method 2 (increase of predictive variance)

### Disadvantages:

- different reaction on higher values of  $a_t^i$  than Method 2
- heuristic as well

# Assigning weights to the data vector $\phi_t$

### Principle:

- significant decrease of the predictive variance (close to 0), setting mean value of regression coefficients to 0
- data are weighted by a<sup>i</sup><sub>t</sub>
- data update

• alternative  $V^{Ai}$ ,  $\nu^{Ai}$  form a proper posterior pdf, keep the computations numerically stable and adjust sensitivity of the method

### Effect:

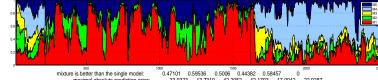
- slower suppression of the respective  $\omega_{t|t}^{i}$  if  $a_{t}^{i} < 1$
- more tolerant
- if  $a_t^i = 1$ , return to the regular operating state is very fast



# Experiments with the method I

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0, 1)$ 





ture is better than the single model:

maximal absolute prediction error:

mean prediction error:

output

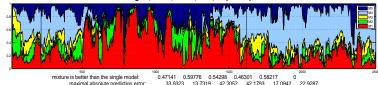
atd. dev. of prediction error:

mean weights of the model:

0.47101 0.59536 0.5006 0.44382 0.58457 0 33.9323 13.7319 4.22052 42.1793 17.0942 22.1 -0.038359 0.014733 0.21971 0.20663 0.01197 -0.008092 0.010899 0.10451 0.26095 0.21509 0.087985 -0.0046642 3.2356 3.5531 3.5137 3.4244 3.5586 3.3036 del: 0.42348 0.13849 0.057111 0.20719 0.17153

V-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0, 1)$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=4

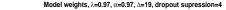


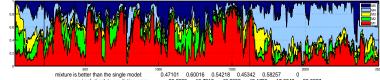
mixture is better than the single model: 0
maximal absolute prediction error:
mean prediction error: -0.0:
median of prediction error: 0.01
std. dev. of prediction error:
mean weights of the model:

0.47141 0.59776 0.54298 0.48301 0.58217 0 0 3.8923 17.0942 22.9287 0.38359 0.014733 -0.046334 -0.086999 0.01197 -0.010759 0.010898 0.10451 -0.0085144 0.004842 0.007895 -0.011168 0.23266 3.5531 3.5915 3.206 3.5586 3.3038 el 0.33472 0.12484 0.08561 0.29215 0.15508

# Experiments with the method II

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.5, 1)$ 

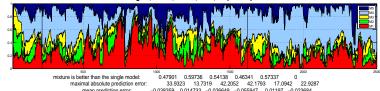




maximal absolute prediction error:
mean prediction error:
-0.038
median of prediction error:
0.010
std. dev. of prediction error:
mean weights of the model:

V-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.5, 1)$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=4



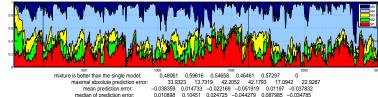
mixture is better than the single model: 0
maximal absolute prediction error:
mean prediction error: -0.0:
median of prediction error: 0.0
std. dev. of prediction error:
mean weights of the model:

0.47901 0.39736 0.94136 0.40341 0.379337 0.38359 0.014733 42.2052 42.1793 17.0942 22.9267 0.38359 0.014733 -0.026649 -0.055947 0.01197 -0.023684 0.10451 0.01524 -0.03961 0.087985 -0.020248 0.32365 3.5351 3.5958 3.3197 3.5586 3.2991

# Experiments with the method III

U-dropout in  $h_1$  between 200 and 1600,  $(b_1, b_2) = (0.9, 1)$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=4

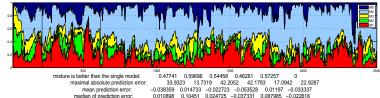


std. dev. of prediction error: mean weights of the model:

3.5961 3.3195 3.5586 3.2933 0.11592 0.44884 0.11972

V-dropout in  $h_1$  between 200 and 600,  $(b_1, b_2) = (0.5, 1)$ 

Model weights,  $\lambda$ =0.97,  $\alpha$ =0.97,  $\Delta$ =19, dropout supression=4



### Advantages:

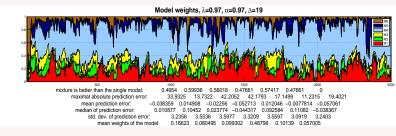
- theoretically based
- fast reactions

### Disadvantages:

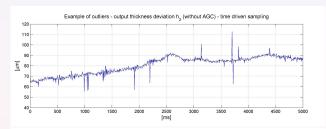
maybe less sensitive?

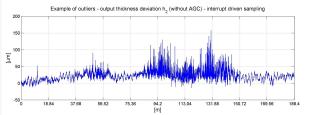
### Another interesting issue

No dropout, 6<sup>th</sup> model containts only one data channel with a sequence -1, 1, -1, 1, -1, ..., i.e. pure nonsense



# Particular corruption of data





### isolated outliers

dirty strip



- corrected wrong likelihood computation → method 1 takes at least small effect
- "emergency" artificial decreasing of D<sub>11</sub> (methods 3 and 4) yields faster recovery than increasing of D<sub>11</sub> (method 2)
- methods 3 and 4 can potentially increase  $D_{11}$ , too, if alternative statistic  $V^A$  has big entries
- nonsense model still admitted
- adaptive median filter in progress
- data scaling and filtration of outliers seems to be necessary

# Thank you for your attention

questions and comments are welcome